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A new sufficient condition for Hamiltonicity of graphs ☆

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Abstract

Rahman and Kaykobad proved the following theorem on Hamiltonian paths in graphs. Let G be a connected graph with n vertices. If $d(u) + d(v) + \delta(u, v) \geqslant n + 1$ for each pair of distinct non-adjacent vertices u and v in G, where $\delta(u, v)$ is the length of a shortest path between u and v in G, then G has a Hamiltonian path. It is shown that except for two families of graphs a graph is Hamiltonian if it satisfies the condition in Rahman and Kaykobad's theorem. The result obtained in this note is also an answer for a question posed by Rahman and Kaykobad.

Keywords: Combinatorial problems; Graphs; Hamiltonian cycles; Hamiltonian paths

We consider only finite undirected graphs without loops and multiple edges. For terminology, notation and concepts not defined here see [2]. A Hamiltonian cycle (resp. path) is a spanning cycle (resp. path) in a graph. A graph G is Hamiltonian (resp. traceable) if G has a Hamiltonian cycle (resp. path). A graph G with n vertices is pancyclic if G has cycles of lengths $3, 4, \ldots, n$. A graph G is Hamilton-connected if for any two distinct vertices u and v there exists a Hamiltonian path between u and v in G. We use $\vee$ to denote the join operator defined in [2]. Sometimes we write a complete graph K_{p+q} with $p + q$ vertices as $K_p \vee K_q$ such that $p \geqslant 1$ and $q \geqslant 0$. A subset S of the vertex set in a graph G is called an independent set of G if no two vertices in S are adjacent in G. An independent set is maximum if G has no independent set T with $|T| > |S|$. The number of vertices, denoted $\alpha(G)$, in a maximum independent set in a graph G is called the independence number of G. For two distinct vertices u and v in a graph G, $\delta(u, v)$ is used to denote the length of a shortest path between u and v in G.

In order to state the main result in this note, we need to define four families of graphs. The first family, denoted $\mathcal{A}_n$, of graphs is defined as the family $\mathcal{G}_n$ of graphs in [1]. The second family, denoted $\mathcal{B}_n$, of graphs consists of a single graph H defined in [1]. For the sake of the completeness, we provide the definitions of $\mathcal{A}_n$ and $\mathcal{B}_n$ here. More detailed descriptions of $\mathcal{A}_n$ and $\mathcal{B}_n$ can be found in [1].

A graph G is in $\mathcal{A}_n$ if and only if $|V(G)| = n$, where $n \geqslant 7$ is an odd integer and the vertex-set of G is the disjoint union of the sets A_1, A_2, B_1, B_2, and $\{a_1, a_2, b\}$ so that

(i) $|A_i \cup B_i| = (n - 3)/2$, $i = 1, 2$,
(ii) $|A_i| \geqslant 2$, $i = 1, 2$,

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